Math 107
Finance Lesson 1: Compound Interest

A 4-year-old stray cat that was rescued from the streets of Rome has inherited a $13 million fortune from its owner, the wealthy widow of an Italian property tycoon.

Maria Assunta left the fortune to her beloved kitty Tommaso when she died two weeks ago at the age of 94. The feline's newfound riches include cash, as well as properties in Rome, Milan and land in Calabria.

As her health began to fail two years ago, Assunta, who had no children, began to seek out a way to see that Tommaso was properly cared for after she died. In November 2009, she bequeathed her entire estate to the alley cat that she'd rescued.

Let's suppose you cat "fluffy" leaves you $10000 in her will – you decide to honor her life by investing this money in an account

All accounts should pay you **interest**. Remember

Interest: Money paid for the use of money

Recall that interest is calculated using the following formula:

I = Interest = Principal x rate x time = Prt

Here **principal** means the original amount invested (sometimes called **present value**) . Rate is the interest rate for the given time period (usually given in years) and time is the time invested (again usually given in years

So the **future value** of an investment (we will use the symbol A to represent this)

A = **future value = P + Prt = Present value + Interest**

Lets suppose you invest the $10000 in an account paying 6% simple interest. How much money will you have after 10 years?

You can just calculate this directly here P = 10000, r = .06 and t = 10

A = 10000 + 10000(.06)(10) = 16000

What is the interest you used on this investment? $6000

You can also do this with Excel – in this case I will look at this investment over the time period. In class we will make a chart like this – pay attention there are some tricks

Year	Balance
0	10000
1	10600
2	11200
3	11800
4	12400
5	13000
6	13600
7	14200
8	14800
9	15400
10	16000

As you can see this is linear growth! Now most investments (certainly all banks) **DO NOT** use simple interest. This as you will see is good for you! Some educational loans and other finance instruments use simple interest, but it pretty rare. More typically banks and other finance institutions will give you **COMPOUND** interest. This means interest is added to your account frequently and the actual balance changes. This was not the case with simple interest!

To see how this works consider fluffy again! – initially you have $10000

After the first year you have $10000 + 10000(.06) or written more compactly $10000(1.06)

The next year you start with $10000(1.06) and at the end of the year you have

$10000(1.06)(1.06) or more compactly $\$10000(1.06)^2$

After three years we would have $\$10000(1.06)^3$

We can generalize this to be $A = P(1 + r)^t$, where P is present value, t is time in years, r is interest rate (annually) and A is future value. But no need to remember the formula, Excel can do it for you – let's see again how this happens

Year	Balance
0	$ 10,000.00
1	$ 10,600.00
2	$ 11,236.00
3	$ 11,910.16
4	$ 12,624.77

5	$	13,382.26
6	$	14,185.19
7	$	15,036.30
8	$	15,938.48
9	$	16,894.79
10	$	17,908.48

But many financial institutions will give you interest more than once per year! Suppose you get an account that pays 6% interest compounded semiannually – this means you get 3% the first six months and then 3% the last six months – how does this work:

Initially = $10000

After six months = $10000 + $10000(.03) or $10000(1.03) or $10000*(1+ .06/2)

After one year = $10000*(1+ .06/2)(1+.06/2)=$10000(1 + .06/2)^2$

After 18 months = $10000(1 + .06/2)^3$

After 2 years = $10000(1 + .06/2)^{2*2}$

And after three years = $10000(1 + .06/2)^{2*3}$

So after t years you get $10000(1 + .06/2)^2$

Do you see that if you had quarterly compounding (4 times a year) you would get: $10000(1 + .06/4)^{4t}$ and monthly compounding you would get $10000(1 + .06/12)^{12t}$

In general if you invest P dollars at r% annually compounded n times per year for t years you get

$$A = P(1 + r/n)^{nt}$$

Now it is hard to remember this formula – so you can do this without the formula using Excel – I will show you how again there are a few tricks!

Excel Example: Suppose your dear old cat "fluffy" leaves you $5000 in her will, you decided to invest the entire amount in an account that pays 8% annual interest, determine the amount you will have after 25 years if
a) You get simple interest
b) You get 8% compounded annually
c) You get 8% compounded semiannually
d) You get 8% compounded monthly
e) You get 8% compounded daily

THE PRESENT VALUE PROBLEM:

In the problem above you had an amount to invest and you wanted to know what is the amount you will have in the future. This is called the future value problem. Instead we sometimes want to know how much we have to invest now to have a certain amount in the future.

Example: Suppose you would like to take a trip for your graduation from college and would like to invest an amount now in an account paying 6% compounded monthly so that you have $2000 in the account in 3 years. How much do you need to invest now?

Note – you can use a formula to do this (there is one above that will work), but again I would like to use Excel to do this problem – you will have to be comfortable with what is know as the “guess and check” method

Practice Example: You would like to have $15000 to help with the purchase of a car in three years. How much would you need ot invest now in a savings account that pays an APR of 9% compounded monthly to reach your goal?

Annual Percentage Yield: Often you want to compare accounts – For example – which is better an account that pays 4.5% compounded monthly or an account the pays 4.6% compounded semiannually. Annual percentage yield gives you the percentage increase on an account that is due to compounding. Another way to say this is what interest rate you would need annually to match the amount you are getting due to compounding more times per year.